CHAPTER 13

ORBITAL MANEUVERS

CHAPTER CONTENT

13- ORBITAL MANEUVERS

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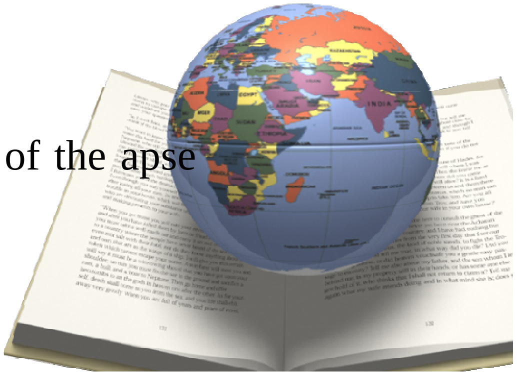
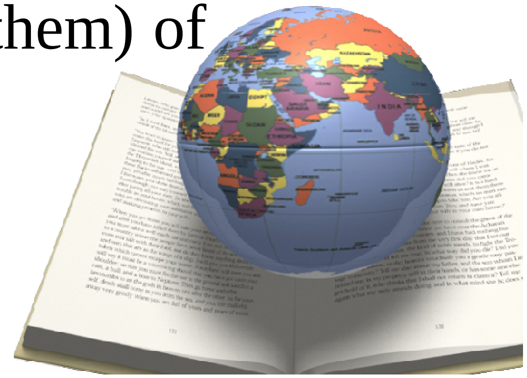
- Orbital maneuvers transfer a spacecraft from one orbit to another.
- Orbital changes can be:
  - The transfer from low-earth parking orbit to an interplanetary trajectory. (Big maneuver)
  - The rendezvous of one spacecraft with another. (Small maneuver)
- Changing orbits required the firing of onboard spacecrafts engines.
- We will use impulsive maneuvers, in which the rockets fire in relatively short bursts to produce the required velocity change Δv
- In this chapter we will consider:
- (NOTE23,P255,{1})
  - Classical, energy-efficient Hohmann transfer maneuvers.
  - The bi-elliptic Hohmann transfer
  - The phasing maneuver. (a from of Hohmann transfer)
  - The non-Hohmann transfer maneuvers with and without rotation of the apse line
  - Chase maneuvers
  - Plane charge maneuvers (introduction)



13- ORBITAL MANEUVERS

- impulsive maneuvers are those in which brief firings of onboard rocket motors change the magnitude and direction of the velocity vector instantaneously.
- During an impulsive maneuver, the position of the spacecraft is considered to be fixed; only the velocity changes.
- (NOTE24,P256,{1})
- Each impulsive maneuver result in a change Δv in the velocity (magnitude , pumping maneuver; direction “cranking maneuver”, or both of them) of spacecraft.



13- ORBITAL MANEUVERS

- The magnitude Δv of the velocity increment is related to Δm, the mass of propellant consumed by the formula

$$\frac{\Delta m}{m} = 1 - e^{-\frac{\Delta v}{I_{sp} g_o}} \qquad (1)$$

m : is the mass of the spacecraft before the burn

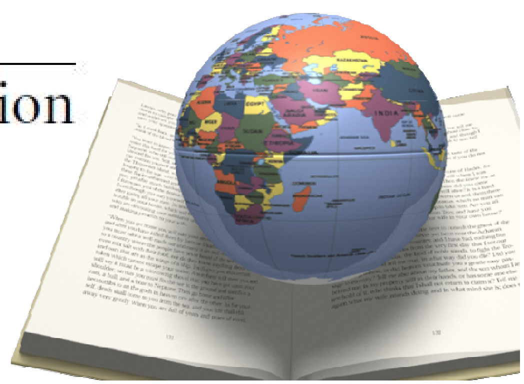
g_o : is the sea-level acceleration of gravity

I_{sp} : is the specific impulse of the propellants.

- Specific impulse is defined as follows:

$$I_{sp} = \frac{\text{thrust}}{\text{sea-level weight rate of fuel consumption}}$$

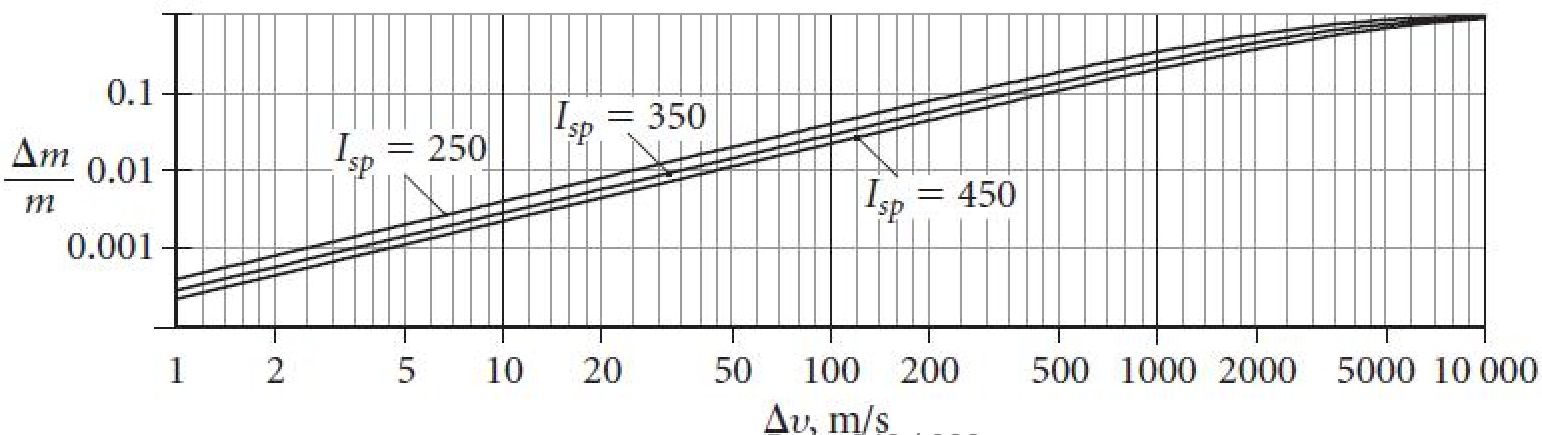
$I_{SP} : [s]$

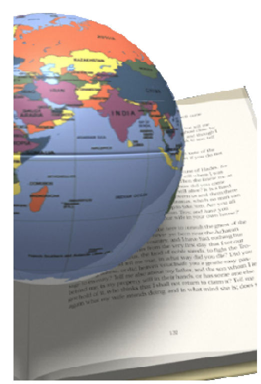


13- ORBITAL MANEUVERS

- I_{sp} for some common propellant combinations are shown in below table:
- (NOTE25,P256,{1})

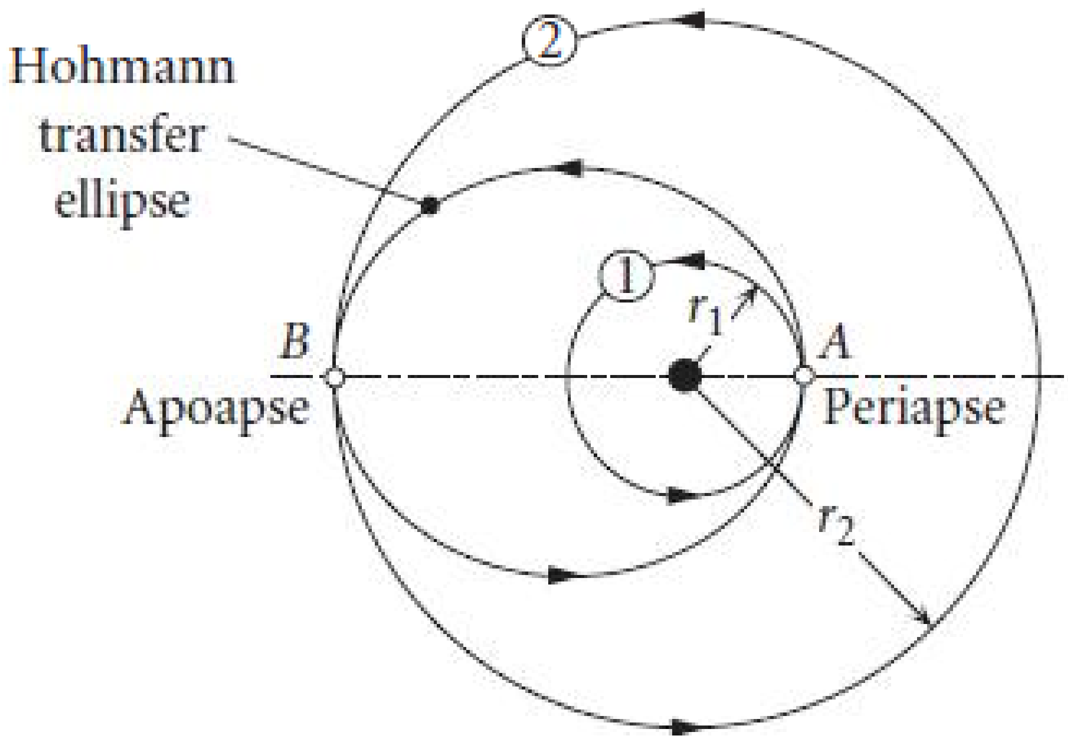
Propellant	I_{sp} (seconds)
Cold gas	50
Monopropellant hydrazine	230
Solid propellant	290
Nitric acid/monomethylhydrazine	310
Liquid oxygen/liquid hydrogen	455

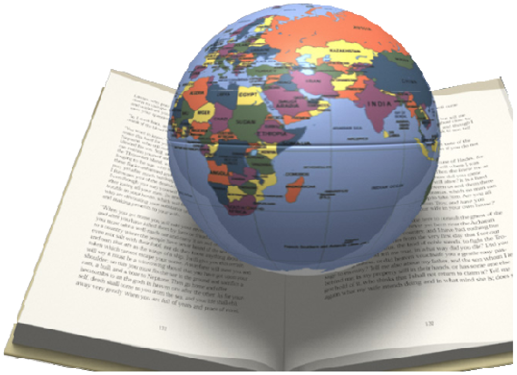




13- HOHMANN TRANSFER

- The Hohmann transfer is the most efficient two-impulse maneuver for transferring between two coplanar circular orbits sharing a common focus.
- (NOTE26,P257,{1})



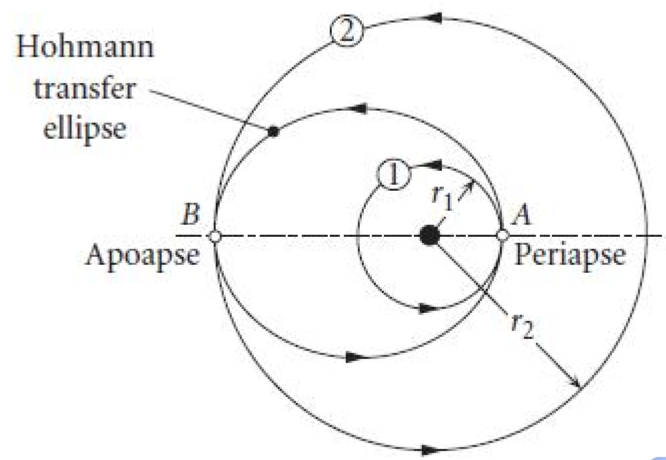


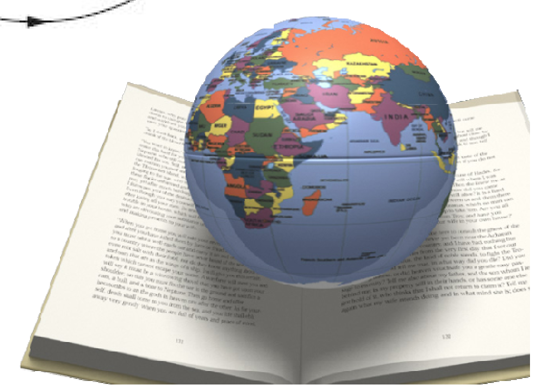
13- HOHMANN TRANSFER

- Recall that for an ellipse the specific energy is negative:

$$\varepsilon = -\frac{\mu}{2a}$$

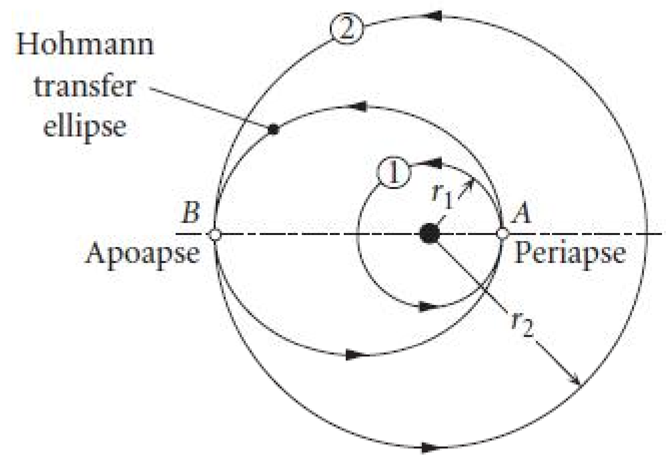
- Increasing the energy requires reducing its magnitude, in order to make ε less negative.
- Therefore, the larger the semimajor axis is, the more the energy the orbit has, the energies increase as we move from the inner to the outer circle.



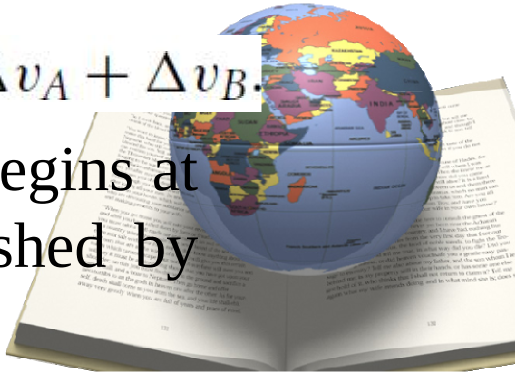


13- HOHMANN TRANSFER

- Starting at A on the inner circle, a velocity increment Δv_A in the direction of flight is required to boost the vehicle onto the higher-energy elliptical trajectory.



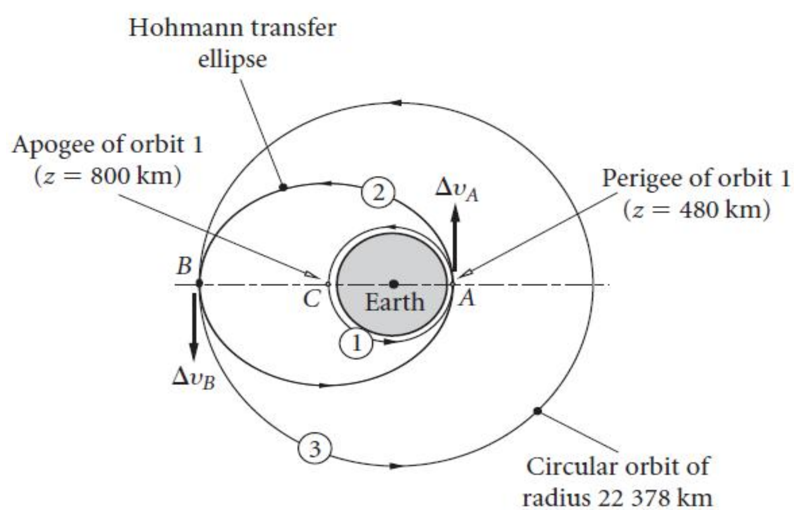
- After coasting from A to B, another forward velocity increment Δv_B places the vehicle on the outer circular orbit.
- The total energy expenditure is: $\Delta v_{\text{total}} = \Delta v_A + \Delta v_B.$
- The same Δv_{total} is required if the transfer begins at B. in this case Δvs must be accomplished by retrofires.

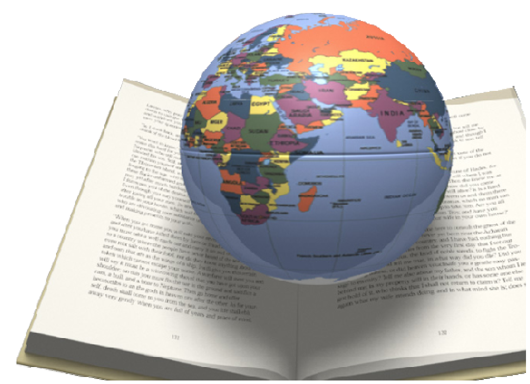


13- HOHMANN TRANSFER

EXAMPLE 13.1

- A spacecraft in a 480km by 800km earth orbit. Find (a) the Δv required at perigee A to place the spacecraft in 480km by 16000km transfer orbit (orbit2); and (b) the Δv (apogee kick) required at B of the transfer orbit to establish a circular orbit of **16000km altitude (orbit3)**





13- HOHMANN TRANSFER

EXAMPLE 13.1

- (a) first, let us establish the primary orbital parameters of the original orbit 1. the perigee and apogee radii are

$$r_A = R_E + z_A = 6378 + 480 = 6858\,\text{km}$$

$$r_C = R_E + z_C = 6378 + 800 = 7178\,\text{km}$$

- Therefore, the eccentricity of orbit 1 is

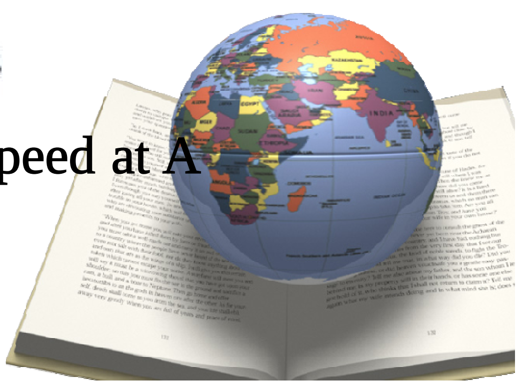
$$e_1 = \frac{r_C - r_A}{r_C + r_A} = 0.022799$$

- Applying the orbit equation at perigee of orbit 1, we calculate the angular momentum,

$$r_A = \frac{h_1^2}{\mu}\frac{1}{1 + e_1\cos(0)} \quad \Rightarrow \quad h_1 = 52\,876\,\text{km}^2/\text{s}$$

- With the angular momentum, we can calculate the speed at A on orbit 1.

$$v_A)_1 = \frac{h_1}{r_A} = 7.7102\,\text{km/s} \qquad \text{(a)}$$



13- HOHMANN TRANSFER

EXAMPLE 13.1

- Moving to the transfer orbit 2, we proceed in a similar fashion to get

$$r_B = R_E + z_B = 6378 + 16\,000 = 22\,378\,\text{km}$$

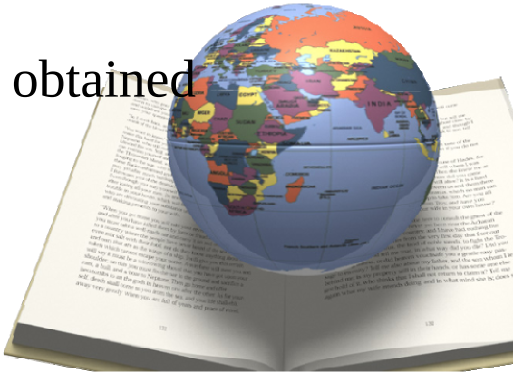
$$e_2 = \frac{r_B - r_A}{r_B + r_A} = 0.53085$$

$$r_A = \frac{h_2^2}{\mu}\frac{1}{1 + e_2\cos(0)} \quad \Rightarrow \quad h_2 = 64\,690\,\text{km}$$

- Thus, the speed at A on orbit 2 is

$$v_A)_2 = \frac{h_2}{r_A} = \frac{64\,690}{6858} = 9.4327\,\text{km/s} \quad \text{(b)}$$

- The required forward velocity increment at A is now obtained from (a) and (b) as



$$\Delta v_A = v_A)_2 - v_A)_1 = \underline{1.7225\,\text{km/s}}$$

13- HOHMANN TRANSFER

EXAMPLE 13.1

- (b) we use angular momentum formula to find the speed at B on orbit 2,

$$v_B)_2 = \frac{h_2}{r_B} = \frac{64\,690}{22\,378} = 2.8908\,\text{km/s} \qquad \text{(c)}$$

- Orbit3 is circular, so its constant orbital speed is obtained from equation ? ,

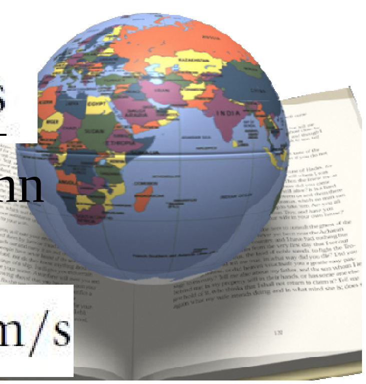
$$v_B)_3 = \sqrt{\frac{398\,600}{22\,378}} = 4.2204\,\text{km/s} \qquad \text{(d)}$$

- Thus, the delta-v requirement at B to climb from orbit 2 to orbit 3 is

$$\Delta v_B = v_B)_3 - v_B)_2 = 4.2204 - 2.8908 = \underline{1.3297\,\text{km/s}}$$

- Observe that the total delta-v requirement for this Hohmann transfer is

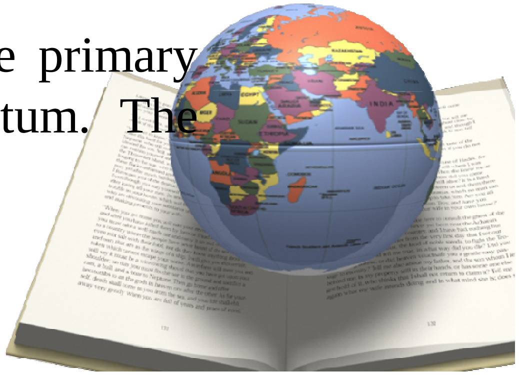
$$\Delta v_{\text{total}} = |\Delta v_A| + |\Delta v_B| = 1.7225 + 1.3297 = 3.0522\,\text{km/s}$$



13- HOHMANN TRANSFER

EXAMPLE 13.2

- A spacecraft returning from a lunar mission approaches earth on a hyperbolic trajectory. At its closest approach A it is an altitude of 5000km, traveling at 10km/s. at A retrorockets are fired to lower the spacecraft into a 500km altitude circular orbit, where it is to rendezvous with a space station. Find the location of the space station at retrofire so that rendezvous will occur at B.
- The time of flight from A to B is one-half the period T_2 of the elliptical transfer orbit 2. while the spacecraft coasts from A to B, the space station coasts through the angle ϕ_{CB} from C to B. Hence, this mission has to be carefully planned and executed, going all the way back to lunar departure, so that the two vehicles meet at B.
- To calculate the period $T_{2;}$, we must first obtain the primary orbital parameters, eccentricity and angular momentum. The apogee and perigee of orbit2, the transfer ellipse, are



$$r_A = 5000 + 6378 = 11\,378\,\text{km}$$

$$r_B = 500 + 6378 = 6878\,\text{km}$$

13- HOHMANN TRANSFER

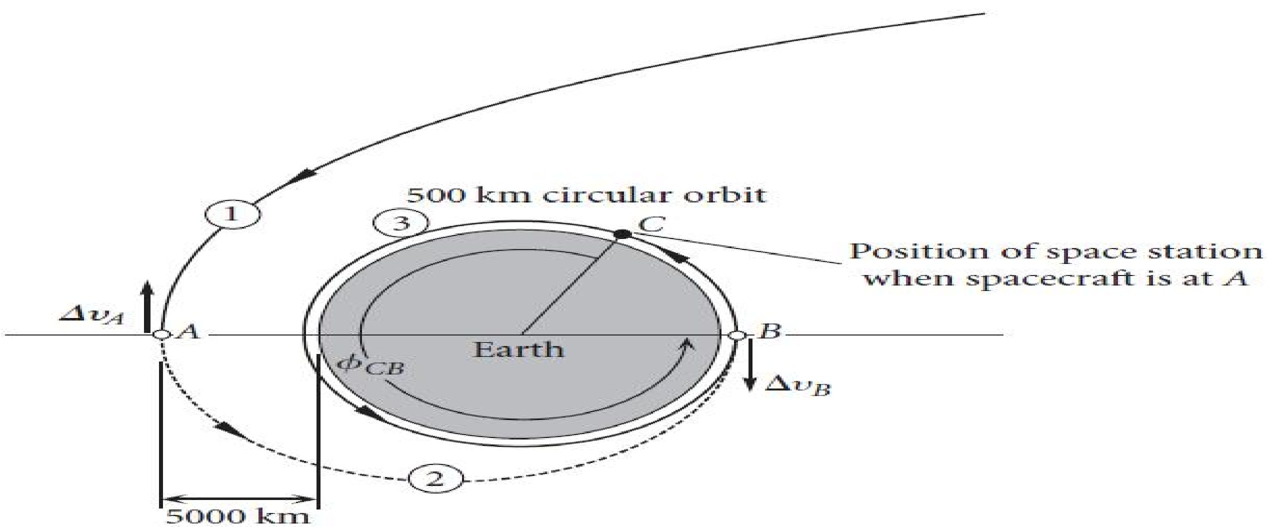
EXAMPLE 13.2

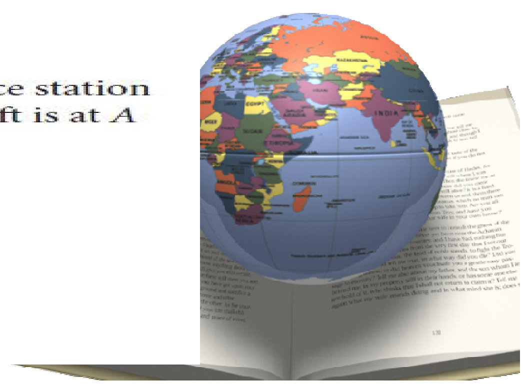
- Therefore, the eccentricity is

$$e_2 = \frac{11\,378 - 6878}{11\,378 + 6878} = 0.24649$$

- Evaluating the orbit equation at perigee yields the angular momentum,

$$r_B = \frac{h_2^2}{\mu}\frac{1}{1+e_2} \quad \Rightarrow \quad 6878 = \frac{h_2^2}{398\,600}\frac{1}{1+0.24649} \quad \Rightarrow \quad h_2 = 58\,458\,\text{km}^2/\text{s}$$





13- HOHMANN TRANSFER

EXAMPLE 13.2

- Now we can use equation ? To find the period of the transfer ellipse,

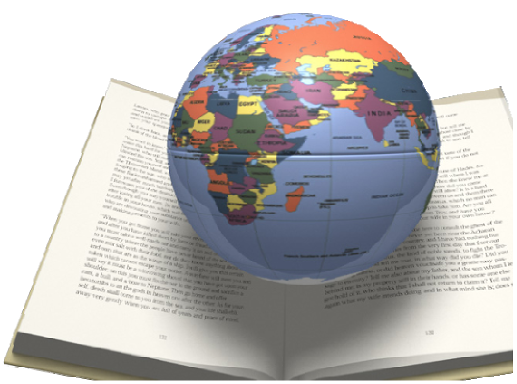
$$T_2 = \frac{2\pi}{\mu^2}\left(\frac{h_2}{\sqrt{1-e_2^2}}\right)^3 = \frac{2\pi}{398\,600^2}\left(\frac{58\,458}{\sqrt{1-0.24649^2}}\right)^3 = 8679.1\text{ s} \quad \text{(a)}$$

- The period of circular orbit3 is, according to equation ?

$$T_3 = \frac{2\pi}{\sqrt{\mu}} r_B^{\frac{3}{2}} = \frac{2\pi}{\sqrt{398\,600}} 6878^{\frac{3}{2}} = 5676.8\text{ s} \quad \text{(b)}$$

- The time of flight from C to B on orbit3 must equal the time of flight from A to B on orbit2.

$$\Delta t_{CB} = \frac{1}{2}T_2 = \frac{1}{2}\cdot 8679.1 = 4339.5\text{ s}$$



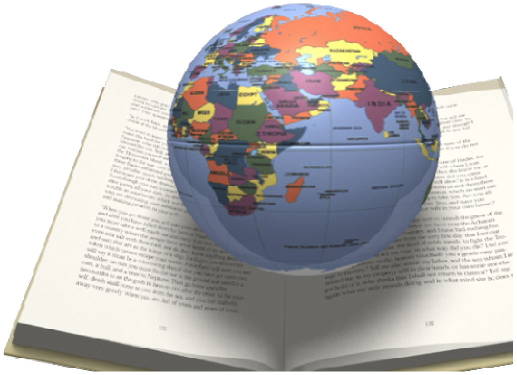
13- HOHMANN TRANSFER

EXAMPLE 13.2

- Since orbit3 is a circle, its angular velocity, unlike an ellipse, is constant. Therefore, we can write

$$\frac{\phi_{CB}}{\Delta t_{CB}} = \frac{360°}{T_3} \Rightarrow \phi_{CB} = \frac{4339.5}{5676.8} \cdot 360 = \underline{275.2°}$$

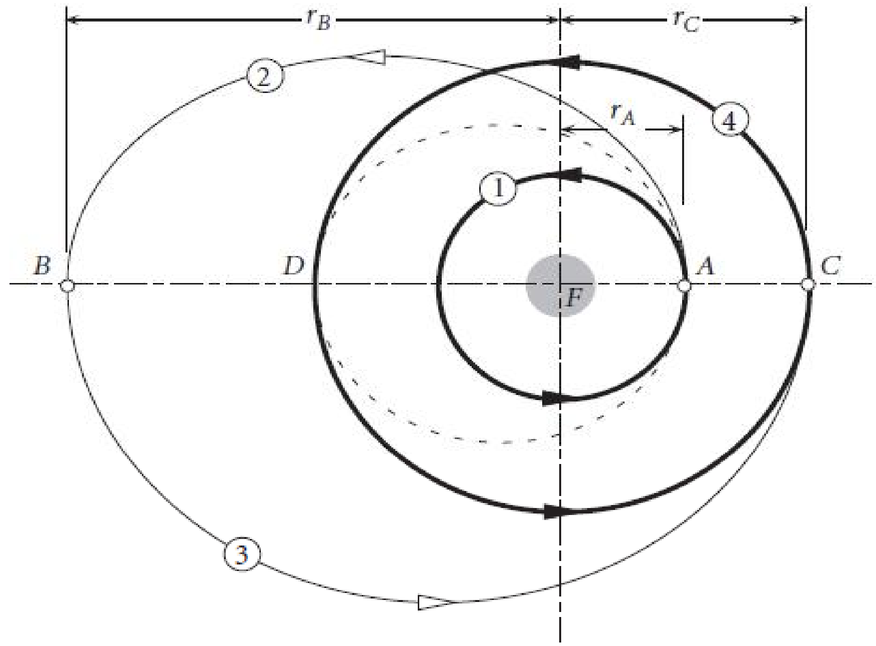
- (the student should verify that the total delta-v required to lower the spacecraft from the hyperbola into the parking orbit is 6.415km/s. A glance at figure ? Reveals the tremendous amount of propellant this would require.)



13- BI-ELLIPTIC HOHMANN TRANSFER

13- BI-ELLIPTIC HOHMANN TRANSFER

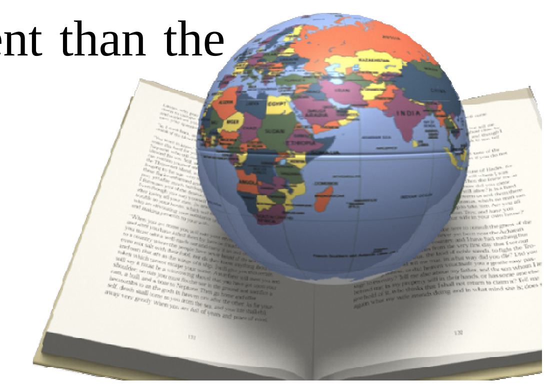
- (NOTE26,P264,{1})
- A Hohmann transfer is the dotted ellips.
- The bi-elliptical Hohmann transfer uses two coaxial semi-ellipses, 2 and 3 (A,B,C)
- The idea is to places B sufficiently far from the focus that the Δv_B will be very small.



$$r_B \longrightarrow \infty \Longrightarrow \Delta v_B \longrightarrow 0$$

- For the bi-elliptical scheme to be more energy efficient than the Hohmann transfer, it must be true that

$$\Delta v_{\text{total}})_{\text{bi-elliptical}} < \Delta v_{\text{total}})_{\text{Hohmann}}$$



13- BI-ELLIPTIC HOHMANN TRANSFER

- Δv analyses of the Hohmann and bi-elliptical transfers lead to the following results:

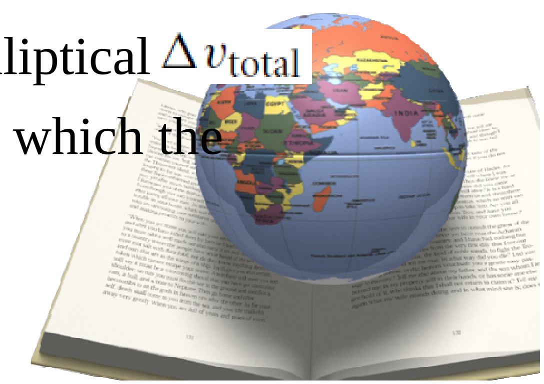
$$\Delta v)_{\text{Hohmann}} = \left[\frac{1}{\sqrt{\alpha}} - \frac{\sqrt{2}(1-\alpha)}{\sqrt{\alpha(1+\alpha)}} - 1\right]\sqrt{\frac{\mu}{r_A}}$$

$$\Delta v)_{\text{bi-elliptical}} = \left[\sqrt{\frac{2(\alpha+\beta)}{\alpha\beta}} - \frac{1+\sqrt{\alpha}}{\sqrt{\alpha}} - \sqrt{\frac{2}{\beta(1+\beta)}}(1-\beta)\right]\sqrt{\frac{\mu}{r_A}}$$

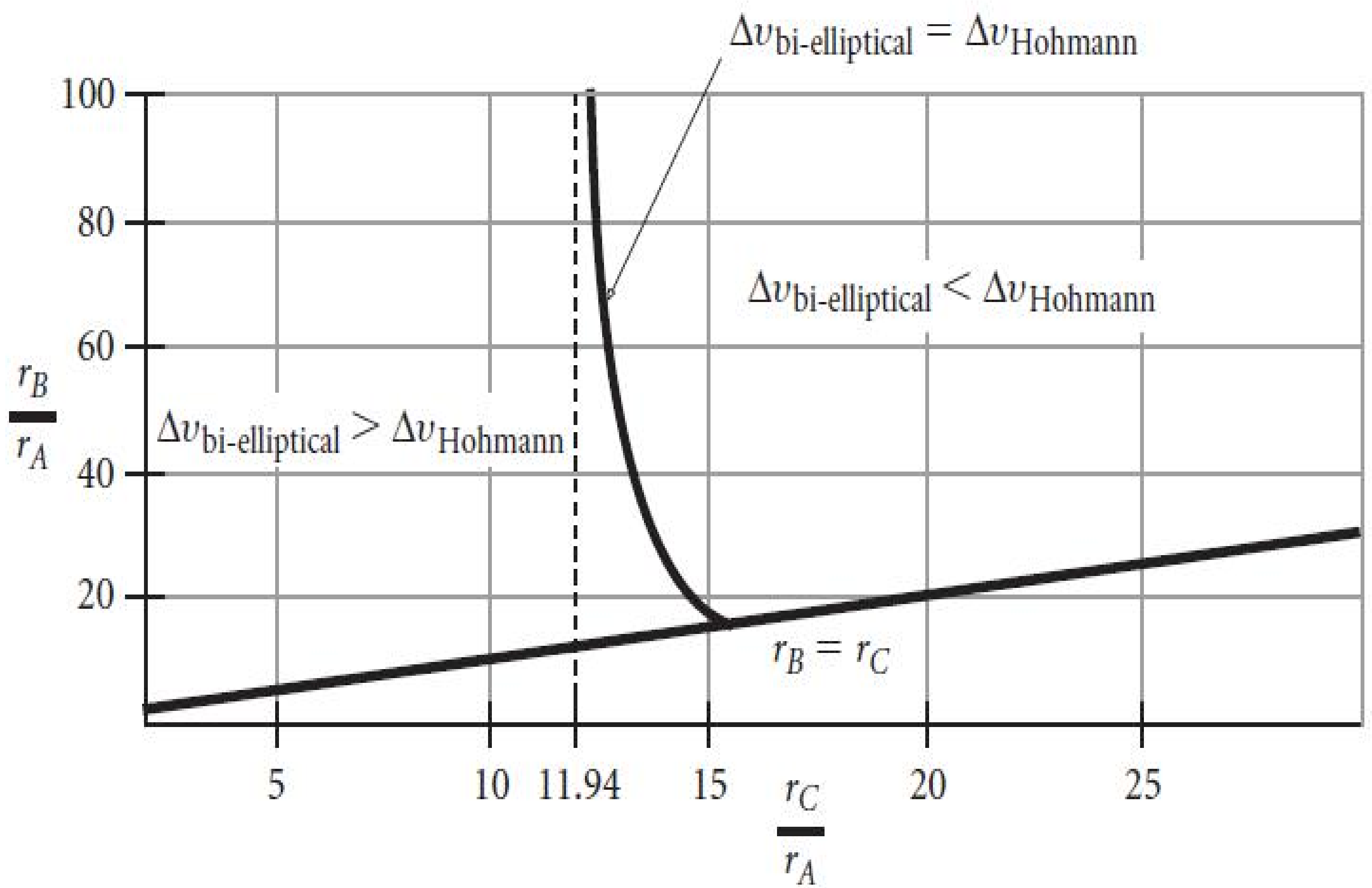
- Where

$$\alpha = \frac{r_C}{r_A} \qquad \beta = \frac{r_B}{r_A}$$

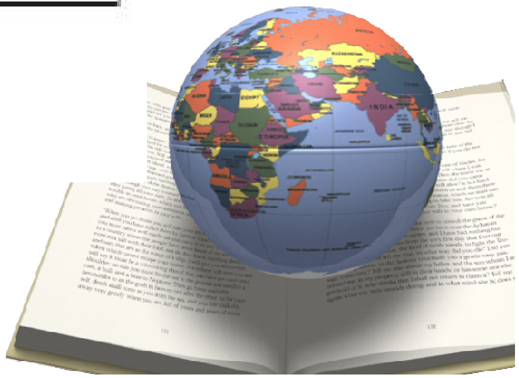
- Plotting the difference between Hohmann and bi-elliptical Δv_{total} as a function of (α) and (β) reveals the regions in which the difference is positive, negative and zero



13- BI-ELLIPTIC HOHMANN TRANSFER



- (NOTE27,P265,{1})



13- BI-ELLIPTIC HOHMANN TRANSFER

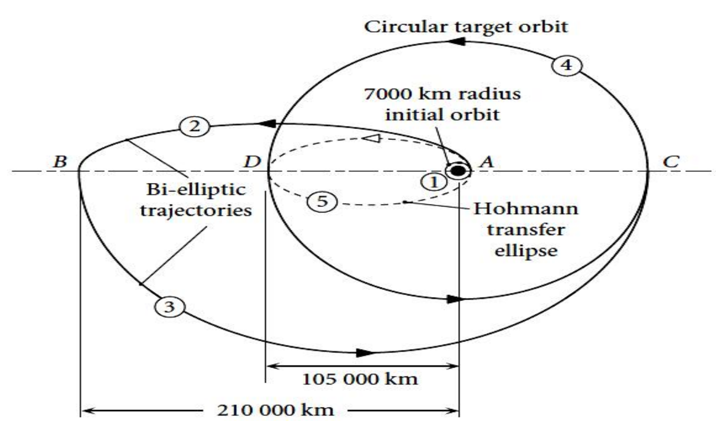
EXAMPLE 13.3

- Find the total delta-v requirement for a bi-ellptical Hohmann transfer from a geocentric circular orbit of 7000km radius to one of 105000km radius. Let the apogee of the first ellipse be 210000km. Compare the delta-v schedule and total flight time with that for an ordinary single Hohmann transfer ellipse.

  Since $r_A = 7000\,\text{km}$ $\quad r_B = 210\,000\,\text{km}$ $\quad r_C = r_D = 105\,000\,\text{km}$

We have $r_B/r_A = 30$ and $r_C/r_A = 15$, so that from the last figure It is apparent right away that the bi-elliptic transfer will be the more energy efficient.

To do the delta-v analysis requires analyzing each of the five orbits.



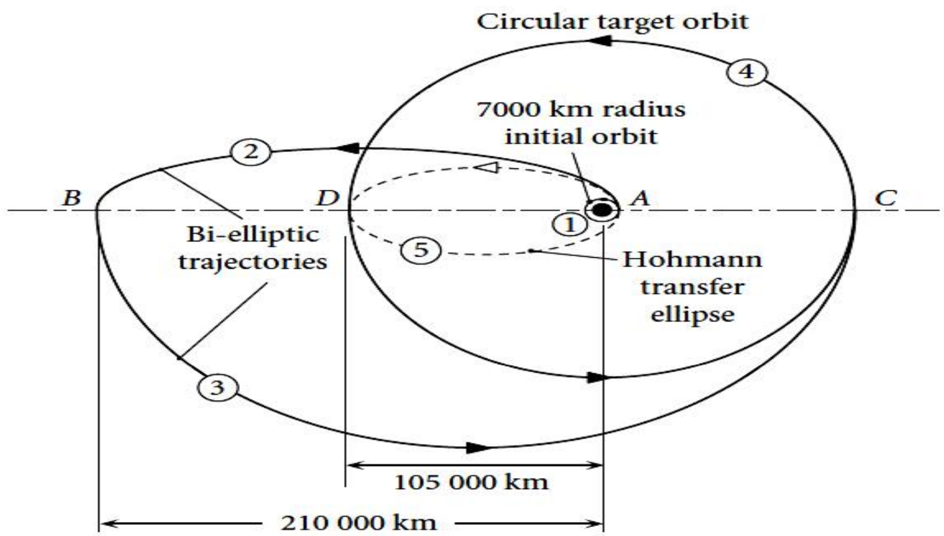
Orbit 1:

Since this is a circular orbit, we have, simply,

$$v_A)_1 = \sqrt{\frac{\mu}{r_A}} = \sqrt{\frac{398\,600}{7000}} = 7.546\,\text{km/s} \qquad \textbf{(a)}$$

13- HOHMANN TRANSFER

EXAMPLE 13.3



- Orbit 2:

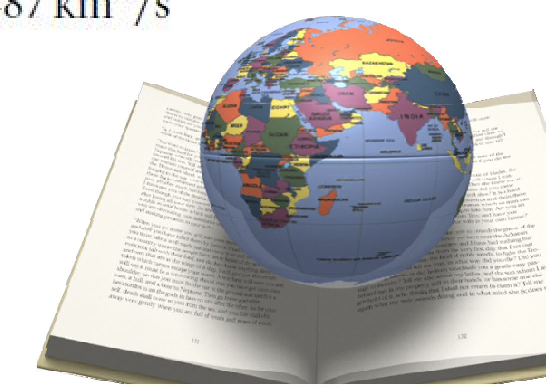
For this transfer ellipse, equation ? yields

$$h_2 = \sqrt{2\mu}\sqrt{\frac{r_A r_B}{r_A + r_B}} = \sqrt{2 \cdot 398\,600}\sqrt{\frac{7000 \cdot 210\,000}{7000 + 210\,000}} = 73\,487\ \text{km}^2/\text{s}$$

- Therefore,

$$v_A)_2 = \frac{h_2}{r_A} = \frac{73\,487}{7000} = 10.498\ \text{km/s} \quad \text{(b)}$$

$$v_B)_2 = \frac{h_2}{r_B} = \frac{73\,487}{210\,000} = 0.34994\ \text{km/s} \quad \text{(c)}$$



13- HOHMANN TRANSFER

EXAMPLE 13.3

- Orbit 3:

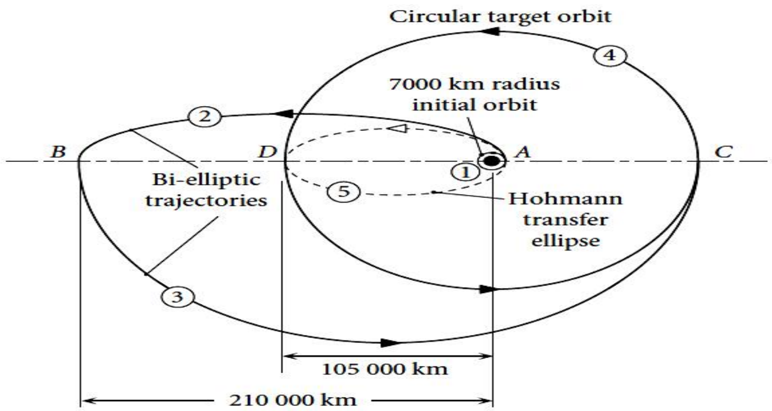
For the second transfer ellipse, we have

$$h_3 = \sqrt{2 \cdot 398\,600}\sqrt{\frac{105\,000 \cdot 210\,000}{105\,000 + 210\,000}} = 236\,230\,\mathrm{km^2/s}$$

- From this we obtain

$$v_B)_3 = \frac{h_3}{r_B} = \frac{236\,230}{210\,000} = 1.1249\,\mathrm{km/s} \qquad \text{(d)}$$

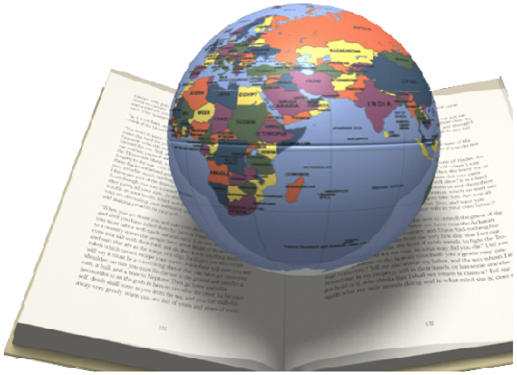
$$v_C)_3 = \frac{h_3}{r_C} = \frac{236\,230}{105\,000} = 2.2498\,\mathrm{km/s} \qquad \text{(e)}$$



- Orbit 4:

The target orbit, like orbit1, is a circle, which means

$$v_C)_4 = v_D)_4 = \sqrt{\frac{398\,600}{105\,000}} = 1.9484\,\mathrm{km/s} \qquad \text{(f)}$$



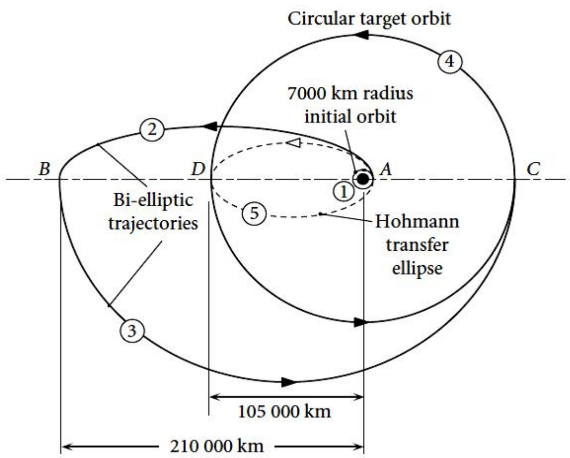
13- HOHMANN TRANSFER

EXAMPLE 13.3

- For the bi-elliptical maneuver, the total delta-v is, therefore,

$$\Delta v_{\text{total}})_{\text{bi-elliptical}} = \Delta v_A + \Delta v_B + \Delta v_C$$
$$= |v_A)_2 - v_A)_1| + |v_B)_3 - v_B)_2| + |v_C)_4 - v_C)_3|$$
$$= |10.498 - 7.546| + |1.1249 - 0.34994| + |1.9484 - 2.2498|$$
$$= 2.9521 + 0.77496 + 0.30142$$

- Or, $\underline{\Delta v_{\text{total}})_{\text{bi-elliptical}} = 4.0285\,\text{km/s}}$ (g)

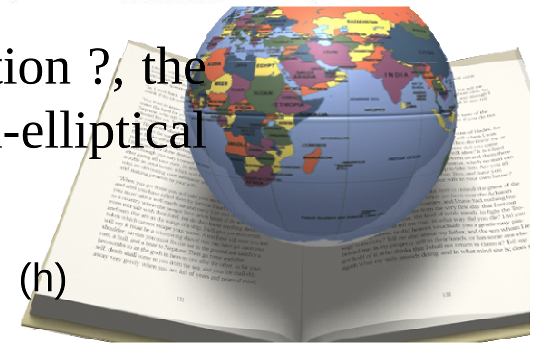


- The semimajor axes of transfer orbits 2 and 3 are

$$a_2 = \frac{1}{2}(7000 + 210\,000) = 108\,500\,\text{km}$$
$$a_3 = \frac{1}{2}(105\,000 + 210\,000) = 157\,500\,\text{km}$$

- With this information and the period formula, equation ?, the time of flight for the two semi-ellipses of the bi-elliptical transfer is found to be

$$t_{\text{bi-elliptical}} = \frac{1}{2}\left(\frac{2\pi}{\sqrt{\mu}}a_2^{\frac{3}{2}} + \frac{2\pi}{\sqrt{\mu}}a_3^{\frac{3}{2}}\right) = 488\,870\,\text{s} = \underline{5.66\,\text{days}} \quad \text{(h)}$$



13- HOHMANN TRANSFER

EXAMPLE 13.3

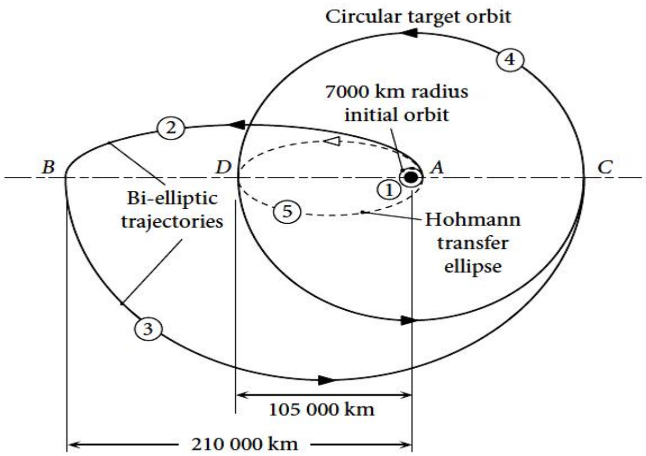
- For the Hohmann transfer ellipse 5,

$$h_5 = \sqrt{2 \cdot 398\,600}\sqrt{\frac{7000 \cdot 105\,000}{7000 + 105\,000}} = 72\,330\ \text{km}^2/\text{s}$$

- Hence,

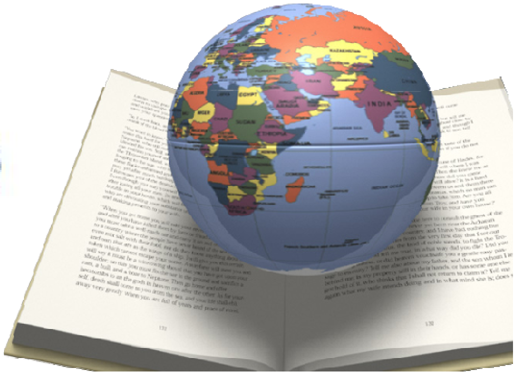
$$v_A)_5 = \frac{h_5}{r_A} = \frac{72\,330}{7000} = 10.333\ \text{km/s} \qquad \text{(i)}$$

$$v_D)_5 = \frac{h_5}{r_D} = \frac{72\,330}{105\,000} = 0.68886\ \text{km/s} \qquad \text{(j)}$$



- It follows that

$$\begin{aligned}\Delta v_{\text{total}})_{\text{Hohmann}} &= |v_A)_5 - v_A)_1| + |v_D)_5 - v_D)_1| \\ &= (10.333 - 7.546) + (1.9484 - 0.68886) \\ &= 2.7868 + 1.2595\end{aligned}$$



13- HOHMANN TRANSFER

EXAMPLE 13.3

- or

$$\underline{\Delta v_{\text{total}})_{\text{Hohmann}} = 4.0463\,\text{km/s}} \qquad \text{(k)}$$

- This is only slightly (0.44 percent) larger than that of the bi-elliptical transfer.

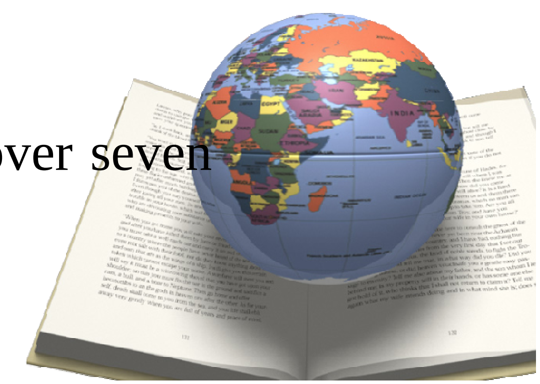
  since the semimajor axis of the Hohmann semi-ellipse is

$$a_5 = \frac{1}{2}(7000 + 105\,000) = 56\,000\,\text{km}$$

- The time of flight from A to D is

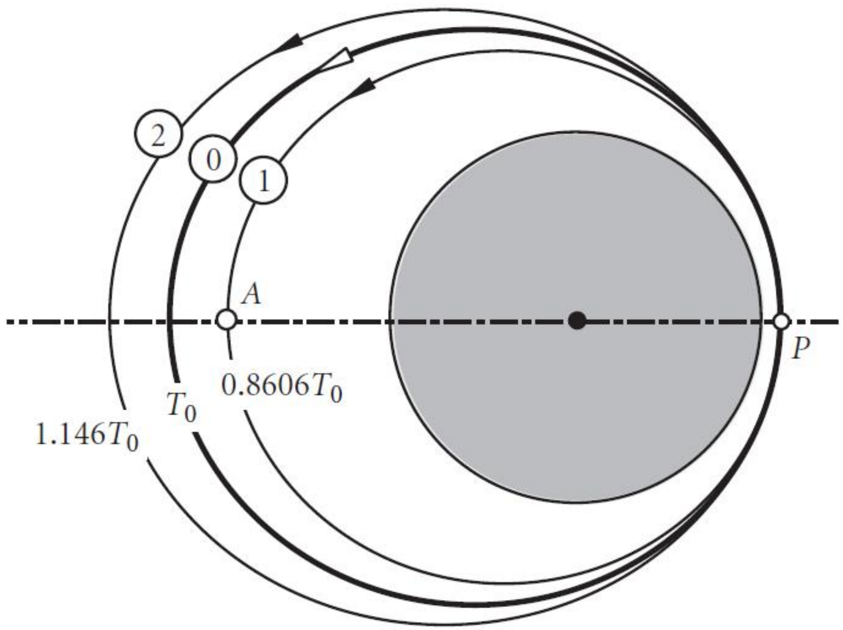
$$t_{\text{Hohmann}} = \frac{1}{2}\left(\frac{2\pi}{\sqrt{\mu}}a_5^{\frac{3}{2}}\right) = 65\,942\,\text{s} = \underline{0.763\,\text{days}} \qquad \text{(l)}$$

- The time of flight of the bi-elliptical maneuver is over seven times longer than that of the Hohmann transfer.

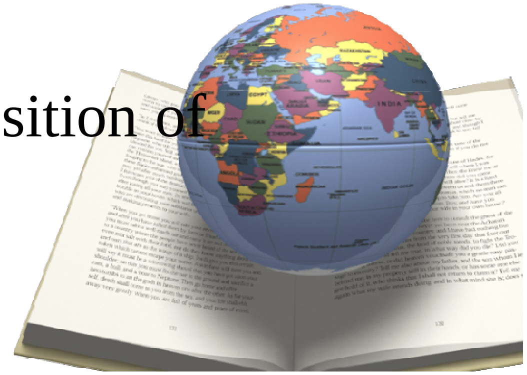


13- PHASING MANEUVERS

- A phasing maneuver is a two-impulse Hohmann transfer from and back to the same orbit, as illustrated in figure:



- Phasing maneuvers are used to change the position of a spacecraft in its orbit.
- (NOTE28,P268,{1}



13- PHASING MANEUVERS

- Once the period T of the phasing orbit is established, then the following equation should be used to determine the semimajor axis of the phasing ellipse:

$$a = \left(\frac{T\sqrt{\mu}}{2\pi}\right)^{\frac{2}{3}} \qquad (5)$$

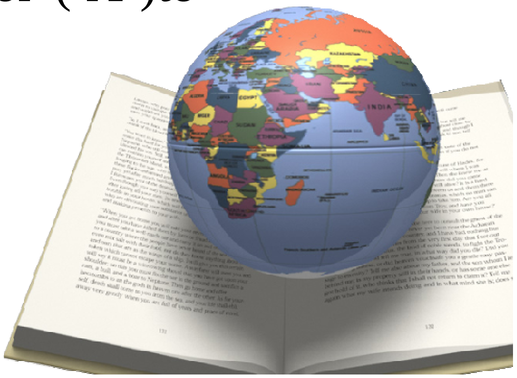
- With the semimajor axis established, r_A opposite to P is obtained from: $2a = r_P + r_A$
- Then we can calculate the eccentricity of phasing orbit from equation:

$$e_3 = \frac{r_B - r_A}{r_B + r_A}$$

- Then the orbit equation may be applied at either (P)or (A)to obtain the angular momentum

$$r = \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta}$$

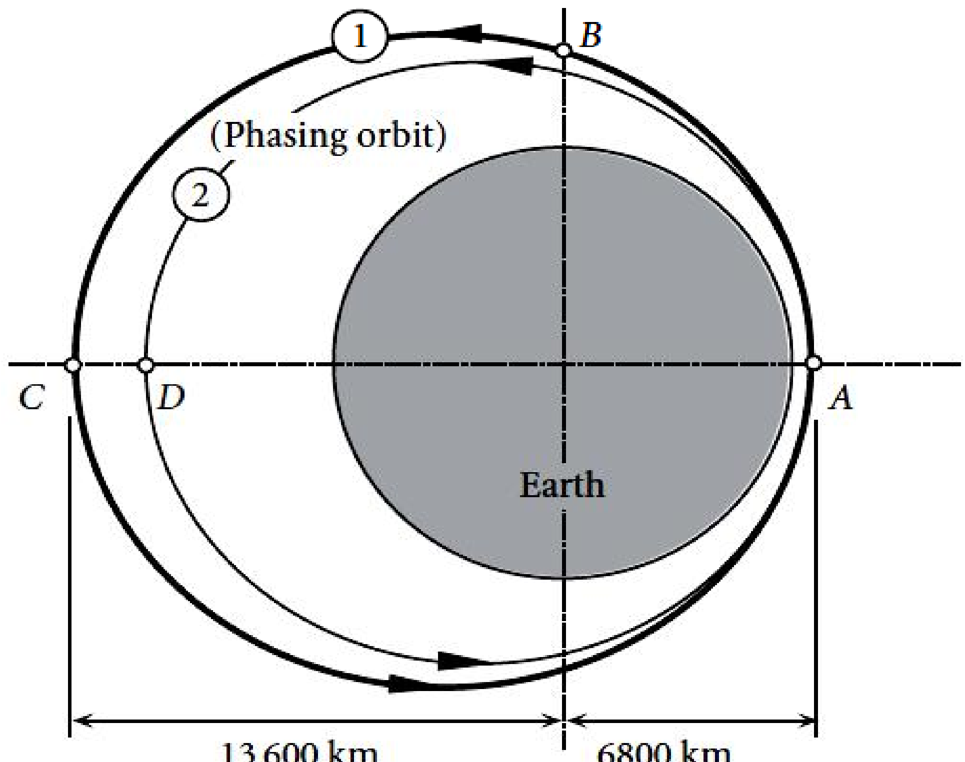
- The phasing orbit is characterized completely

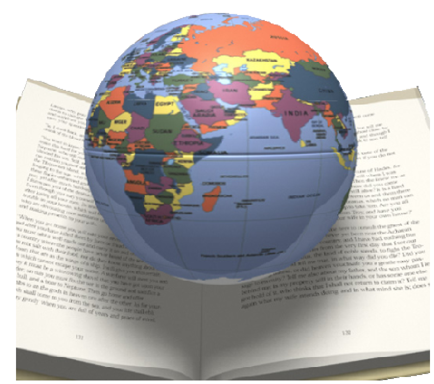


13- PHASING MANEUVERS

EXAMPLE 13.4

* Spacecrafts at A and B are in the same orbit (1). At the instant shown, the chaser vehicle at A executes a phasing maneuver so as to catch the target spacecraft back at A after just one revolution of the chaser's phasing orbit (2). What is the required total delta-v?





13- PHASING MANEUVERS

EXAMPLE 13.4

- From the figure, $r_A = 6800\,\text{km} \qquad r_C = 13\,600\,\text{km}$
- Orbit 1:

The eccentricity of orbit 1 is $e_1 = \dfrac{r_C - r_A}{r_C + r_A} = 0.33333$

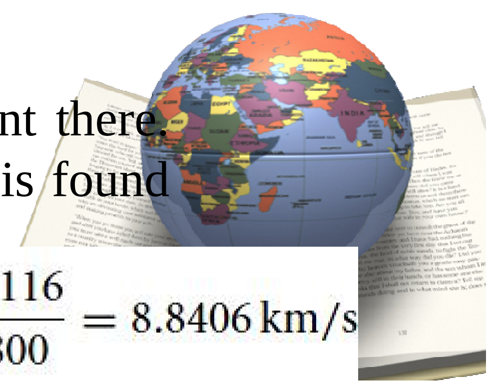
Evaluating the orbit equation at A, we find

$$r_A = \frac{h_1^2}{\mu}\frac{1}{1 + e_1\cos(0)} \Rightarrow 6800 = \frac{h_1^2}{398\,600}\frac{1}{1 + 0.3333} \Rightarrow h_1 = 60\,116\,\text{km}^2/\text{s}$$

The period is found using equation ?

$$T_1 = \frac{2\pi}{\mu^2}\left(\frac{h_1}{\sqrt{1 - e_1^2}}\right)^3 = \frac{2\pi}{398\,600^2}\left(\frac{60\,116}{\sqrt{1 - 0.33333^2}}\right)^3 = 10\,252\,\text{s}$$

- Since A is perigee, there is no radial velocity component there. The speed, directed entirely in the transverse direction, is found from the angular momentum formula,



$$v_{A1} = \frac{h_1}{r_A} = \frac{60\,116}{6800} = 8.8406\,\text{km/s}$$

13- PHASING MANEUVERS

EXAMPLE 13.4

The phasing orbit must have a period T_2 equal to the time it takes the target vehicle at B to coast around to point A on orbit 1. we can determine the flight time by calculating Δt_{AB} time from A to B and subtracting that result from the period T_1 of orbit 1. At B the true anomaly is $\theta_A = 90°$. herefore, according to equation ?

$$\tan\frac{E_B}{2} = \sqrt{\frac{1-e_1}{1+e_1}}\tan\frac{\theta_B}{2} = \sqrt{\frac{1-0.33333}{1+0.33333}}\tan\frac{90°}{2}$$

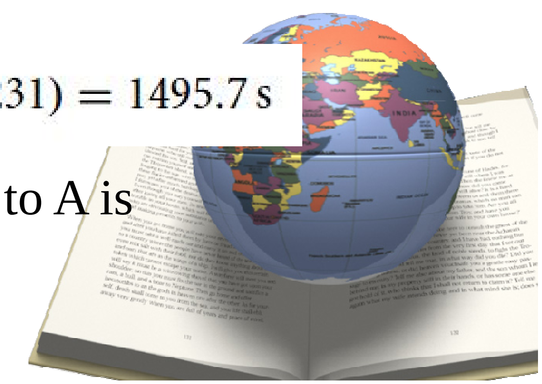
$$= 0.70711 \Rightarrow E_B = 1.2310\,\text{rad}$$

- Then, from Kepler's equation (?), we get

$$\Delta t_{AB} = \frac{T_1}{2\pi}(E_B - e_1 \sin E_B) = \frac{10\,252}{2\pi}(1.231 - 0.33333 \cdot \sin 1.231) = 1495.7\,\text{s}$$

- Thus, the time of flight of the target spacecraft from B to A is

$$\Delta t_{BA} = T_1 - \Delta t_{AB} = 10\,252 - 1495.7 = 8756.3\,\text{s}$$



13- PHASING MANEUVERS

EXAMPLE 13.4

- Orbit 2:

  The period of orbit 2 must equal Δt_{BA} so that the chaser will arrive at A when the target does. That is,

$$T_2 = 8756.3\,\text{s}$$

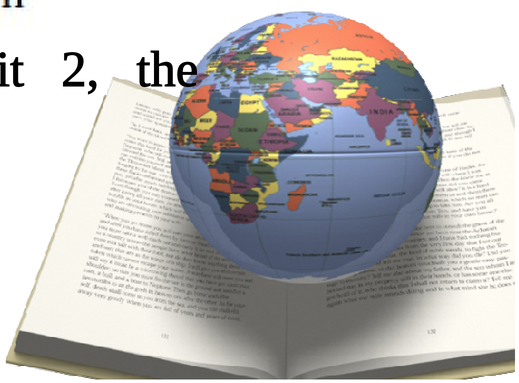
This, together with the period formula, equation ?, yields the semimajor axis of orbit 2,

$$T_2 = \frac{2\pi}{\sqrt{\mu}} a_2^{\frac{3}{2}} \Rightarrow 8756.2 = \frac{2\pi}{\sqrt{398\,600}} a_2^{\frac{3}{2}} \Rightarrow a_2 = 9182.1\,\text{km} \qquad \text{(a)}$$

Since $2a_2 = r_A + r_D$, we find

$$r_D = 2a_2 - r_A = 2 \cdot 9182.1 - 6800 = 11\,564\,\text{km}$$

Therefore, point A is indeed the perigee of orbit 2, the eccentricity of which can now be determined:



$$e_2 = \frac{r_D - r_A}{r_D + r_A} = 0.25943$$

13- PHASING MANEUVERS

EXAMPLE 13.4

Evaluating the orbit equation at point A orbit 2 yields its angular momentum,

$$r_A = \frac{h_2^2}{\mu}\frac{1}{1+e_2\cos(0)} \Rightarrow 6800 = \frac{h_2^2}{398\,600}\frac{1}{1+0.25943} \Rightarrow h_2 = 58\,426\,\mathrm{km^2/s}$$

Finally, we can calculate the speed at perigee of orbit **2,**

$$v_{A_2} = \frac{h_2}{r_A} = \frac{58\,426}{6800} = 8.5921\,\mathrm{km/s}$$

At the beginning of the phasing maneuver,

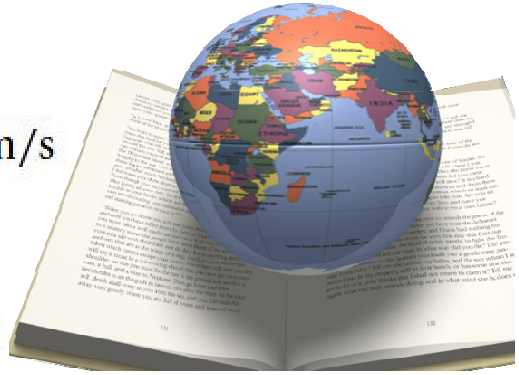
$$\Delta v_A = v_{A_2} - v_{A_1} = 8.5921 - 8.8406 = -0.24851\,\mathrm{km/s}$$

At the end of the phasing maneuver,

$$\Delta v_A = v_{A_1} - v_{A_2} = 8.8406 - 8.5921 = 0.24851\,\mathrm{km/s}$$

The total delta-v, therefore, is

$$\Delta v_{\text{total}} = |-0.24851| + |0.24851| = \underline{0.4970\,\mathrm{km/s}}$$



13- PHASING MANEUVERS

EXAMPLE 13.5

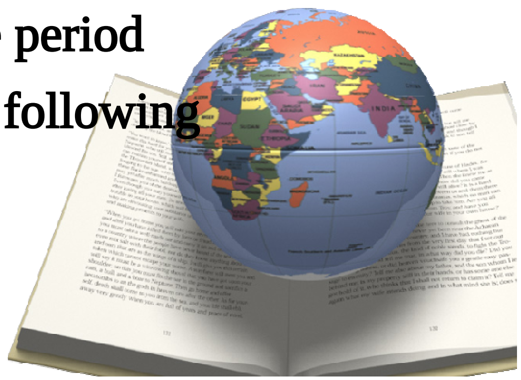
- It is desired to shift the longitude of a GEO satellite $12°$ westward in three revolutions of its phasing orbit. Calculate the delta-v requirement.

  this problem is illustrated in ? . It may be recalled from equation ?, ? and ? That the angular velocity of the earth, the radius to GEO and the speed in GEO are, respectively,

$$\omega_E = \omega_{\text{GEO}} = 72.922 \times 10^{-6} \text{ rad/s}$$
$$r_{\text{GEO}} = 42\,164 \text{ km} \qquad \textbf{(a)}$$
$$v_{\text{GEO}} = 3.0747 \text{ km/s}$$

- **Let $\Delta\Lambda$ be the change in longitude in radians. Then the period T_2 of the phasing orbit can be obtained from the following formula,**

$$\omega_E(3T_2) = 3 \cdot 2\pi + \Delta\Lambda \qquad \textbf{(b)}$$

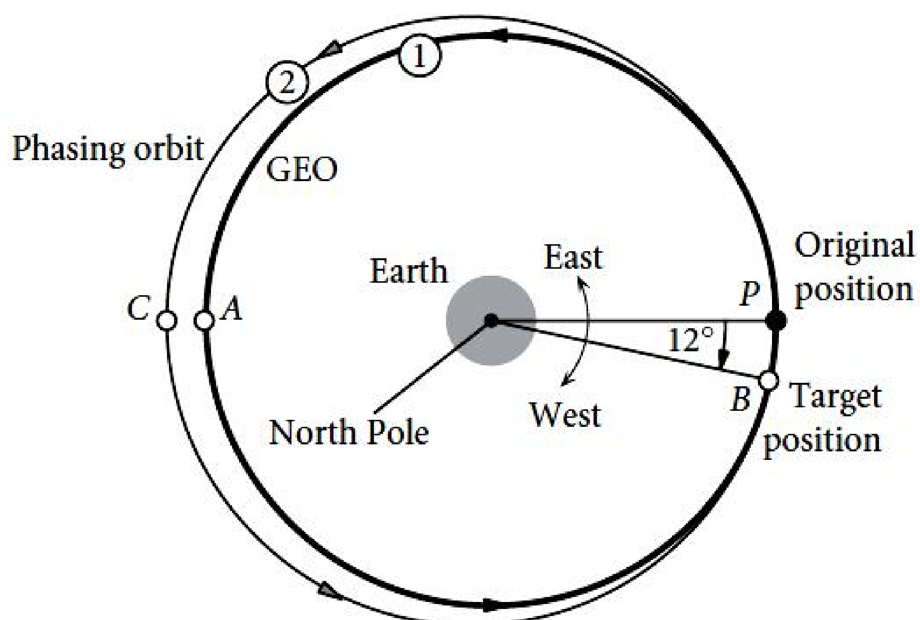


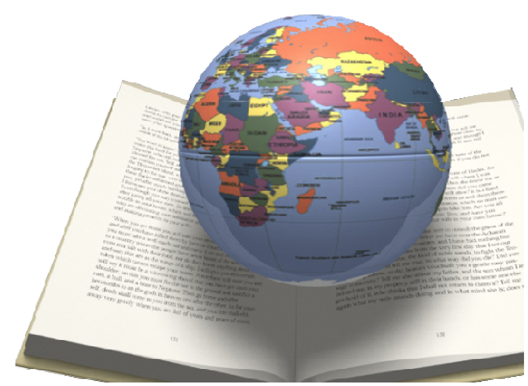
13- PHASING MANEUVERS

EXAMPLE 13.5

which states that after three circuits of the phasing orbit, the original position of the satellite will be $\Delta\Lambda$ radians east of P. in other words, the satellite will end up $\Delta\Lambda$ radians west of its original position in GEO, as desired. From (b) we obtain,

$$T_2 = \frac{1}{3}\frac{\Delta\Lambda + 6\pi}{\omega_E} = \frac{1}{3}\cdot\frac{12^\circ \cdot \frac{\pi}{180^\circ} + 6\pi}{72.922 \times 10^{-6}} = 87\,121 \text{ s}$$





13- PHASING MANEUVERS

EXAMPLE 13.5

Note that the period of GEO is

$$T_{GEO} = \frac{2\pi}{\omega_{GEO}} = 86\,163\ \text{s}$$

The satellite in its slower phasing orbit appears to drift westward at the rate

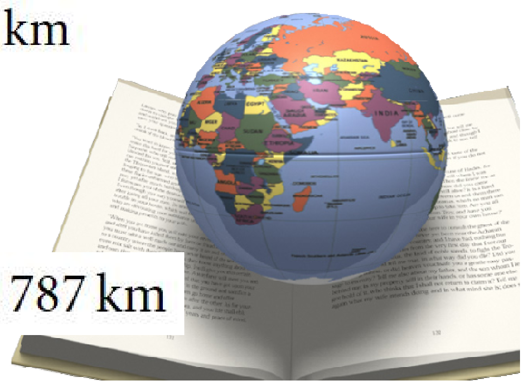
$$\dot{\Lambda} = \frac{\Delta\Lambda}{3T_2} = 8.0133 \times 10^{-7}\ \text{rad/s} = 3.9669°/\text{day}$$

Having the period, we can use equation ? To obtain the semimajor axis of orbit 2,

$$a = \left(\frac{T\sqrt{\mu}}{2\pi}\right)^{\frac{2}{3}} = \left(\frac{87\,121\sqrt{398\,600}}{2\pi}\right)^{\frac{2}{3}} = 42\,476\ \text{km}$$

From this we find the radial coordinate of C,

$$2a_2 = r_P + r_C \;\Rightarrow\; r_C = 2 \cdot 42\,476 - 42\,164 = 42\,787\ \text{km}$$



13- PHASING MANEUVERS

EXAMPLE 13.5

Now we can find the eccentricity of orbit 2,

$$e_2 = \frac{r_C - r_A}{r_C + r_A} = \frac{42\,787 - 42\,164}{42\,787 + 42\,164} = 0.0073395$$

And the angular momentum follows from applying the orbit equation at P (or C) of orbit 2:

$$r_P = \frac{h_2^2}{\mu}\frac{1}{1 + e_2\cos(0)} \Rightarrow 42\,164 = \frac{h_2^2}{398\,600}\frac{1}{1 + 0.0073395} \Rightarrow h_2 = 130\,120\,\text{km}^2/\text{s}$$

at P the speed in orbit 2 is

$$v_{P_2} = \frac{130\,120}{42\,164} = 3.0859\,\text{km/s}$$

therefore, at the beginning of the phasing orbit,

$$\Delta v = v_{P_2} - v_{\text{GEO}} = 3.0859 - 3.0747 = 0.01126\,\text{km/s}$$

at the end of the phasing maneuver,

$$\Delta v = v_{\text{GEO}} - v_{P_2} = 3.0747 - 3.08597 = -0.01126\,\text{km/s}$$

Therefore, $\Delta v_{\text{total}} = |0.01126| + |-0.01126| = \underline{0.022525\,\text{km/s}}$